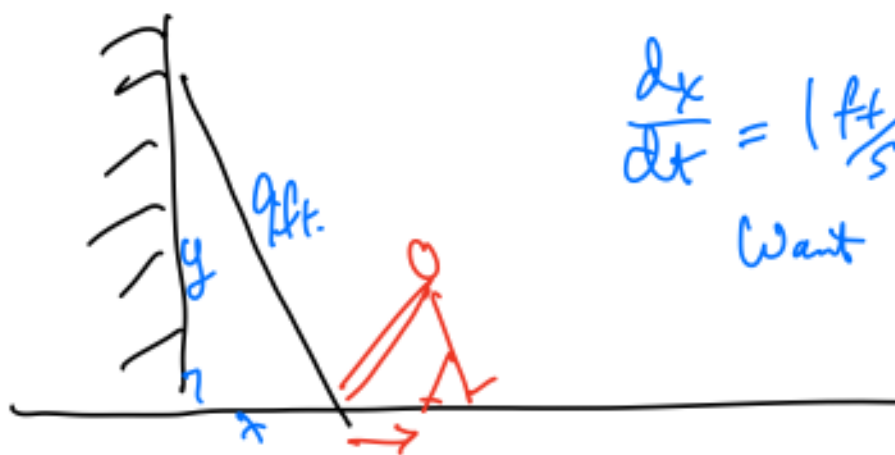


## Our first example:

A 9 foot ladder is leaning up against a wall. Aria pulls the bottom of the ladder away from the wall at a rate of 1 ft per second. How fast is the top of the ladder moving down the wall when the bottom is

- (a) 2 ft from the wall?
- (b) 7 ft from the wall?



$$\frac{dx}{dt} = 1 \frac{\text{ft}}{\text{s}}$$

$$\text{Want } -\frac{dy}{dt} = ?$$

$$x^2 + y^2 = 81$$

Take derivative:

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

$$2x \frac{dx}{dt} = -2y \frac{dy}{dt}$$

$$\frac{x}{y} \frac{dx}{dt} = -\frac{dy}{dt}$$

$$(a) \quad x=2, \quad \frac{dx}{dt}=1, \quad y = \sqrt{81-x^2} = \sqrt{81-4} = \sqrt{77}$$

$$\Rightarrow -\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt} = \frac{2}{\sqrt{77}} \cdot 1 = \boxed{0.2279 \text{ ft/s}}$$

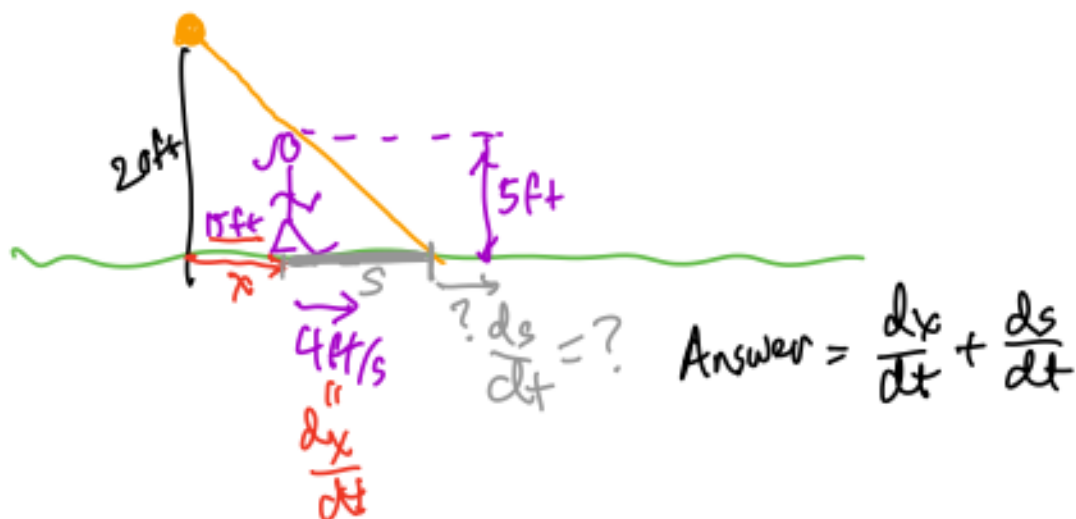
(b)  $x = 7$ ,  $\frac{dx}{dt} = 1$ ,  $y = \sqrt{81 - x^2} = \sqrt{81 - 49} = \sqrt{32}$

$$\Rightarrow -\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt} = \frac{7}{\sqrt{32}} \cdot 1 = \boxed{1.2374 \text{ ft/s}}$$

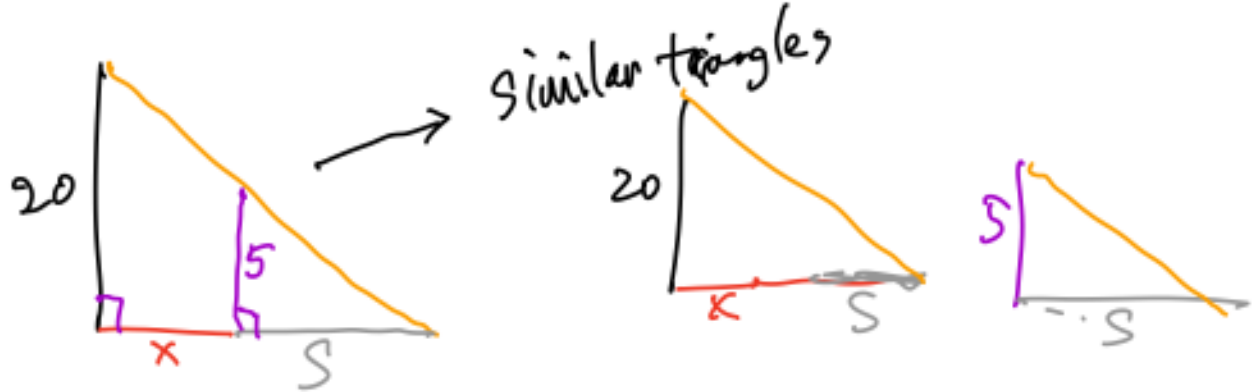
(c) When  $x = 8.9$  ft,  $\frac{dx}{dt} = 1$ ,  $y = \sqrt{81 - (8.9)^2} = \sqrt{81 - (79.21)} = \sqrt{1.79}$

$$\Rightarrow -\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt} = \frac{8.9}{\sqrt{1.79}} \cdot 1 = \boxed{6.6521 \text{ ft/s}}$$

**Example** A streetlight is 20 ft high, and Olivia walks away from the street light at a rate of 4 ft/s. Olivia is 5 ft tall. How fast is Olivia's shadow moving when Olivia is 15 ft from the street light? Will she catch it?



Need equation relating  $x$  &  $s$



$$\Rightarrow \frac{x+s}{20} = \frac{s}{5}$$

big triangle      small triangle

$$\Rightarrow x + s = 4s$$

$$x = 3s$$

derivative:

$$\frac{dx}{dt} = 3 \frac{ds}{dt}$$

$$\frac{dx}{dt} = \frac{4ft}{s} = 3 \frac{ds}{dt}$$

$$\Rightarrow \frac{4}{3} \frac{ft}{s} = \frac{ds}{dt}$$

Ans:

$$\frac{dx}{dt} + \frac{ds}{dt} = \frac{4ft}{s} + \frac{4}{3} \frac{ft}{s}$$

$$= \boxed{5 \frac{1}{3} \frac{ft}{s}}$$

**Example** A laser flashlight is hanging from a ceiling and is rotating at a rate of 5 revolutions per second. It is at a height of 7 ft, at the center of a room that is 12 ft  $\times$  20 ft. How fast is the flashlight's beam moving when it is at 1 ft from the corner of one of the 12 ft walls?

